

Basic Ideas and Concepts for Multiple Comparison Procedures

Kei Takeuchi

Around the year 1970, the author proposed a basic framework for multiple decision procedures including MCP's.

Let X be a random variable and a set of probability distributions indexed by θ in a set of Ω is given. We have a family S of subsets ω_α indexed by α , and we want to decide to which ω_α belongs to. We assume that all θ belongs to some ω_α , not necessarily to only one of them. We assume for two different ω_α and $\omega_{\alpha'}$, neither one is a subset of another. A function ψ of X which designate a subset B of A corresponding to X is called a multiple decision procedure (MDP) with confidence level $1-\alpha$, when for all $\theta \in \Omega$, $\psi(X)$ includes the true θ with probability not smaller than $1-\alpha$. Then correspondingly, for each $\alpha \in A$ we define $\phi_\alpha(X)$ as $\phi_\alpha(X) = 1$ if $\psi(X)$ does not include ω_α , and $= 0$ otherwise, we have a test function for the composite hypothesis that θ belong to ω_α . Conversely if we have a system of tests corresponding to the hypothesis that θ belongs to ω for all α , we can derive a MDP with confidence level $1-\alpha$, by collecting all ω_α for which the hypothesis that θ belongs that ω_α is accepted.

Thus a MDP and a system of tests correspond each other, and obtaining MCP can be reduced to constructing systems of testing composite hypotheses. Then we have to consider two criteria: 1) powers of each test, 2) coziness of the derived MDP. Several examples will be discussed and it will be shown that the system of likelihood tests often leads to a complicated awkward MDP, and other tests seem to be more appealing.