Distribution of A Random Geometric Coverage Problem

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Abstract

Statistical simulation is an important tool because many problems in statistics have no analytic solutions. Motivated by a random coverage problem from a real-life case study on the distribution of overlapping area between a fixed circle and a set of m random circles Fang and Wang (1994) applied the number-theoretic method for related statistical simulation.

Let $B_2 = \{(x, y) : x^2 + y^2 \le 1\}$ be the unit circle. Suppose that there are m random circles $O_1, \cdots, O_m$ with centers and radii $P_1, \cdots, P_m$ and $R_1, \cdots, R_m$, respectively and that each P_i follows a bivariate normal distribution, $P_i \sim N_2(0, \sigma_i^2 I_2), 1 < i < m$. Let S be the overlapping area between B_2 and the union of all random circles, i.e. $S = B_2 \cap (O_1 \cup \cdots \cup O_m)$.Denote the area of S by A_s. The goal is to find the distribution of A_s.

It is easily to find out the distribution of S if $m = 1$, since the overlapping area of two circles can be expressed explicitly by the distribution of their centers and the radii of these two circles. When $m > 1$, it is difficult to find out a simple formula for the distribution of S. Therefore, a natural way is to use the method of statistical simulation. In this talk we discuss and compare several number-theoretic methods including the equi-lattice point (ELP net), glp method (NT net), and several other methods in construction of statistical simulations (Hua and Wang (1981)) Some theoretic and simulation results are given.

Reference

[1] Fang, K. T. and Wang, Y. (1994), *Number-Theoretic Methods in Statistics*, Chapman and Hall, London.

[2] Hua, L. K. and Wang, Y. (1981), *Applications of Number Theory to Numberical Analysis*, Springer Verlag and Science Press.