On Identification of Inferior Treatments Using the Newman-Keuls Procedure

Samuel S. Wu
University of Florida
Malcom Randall VA Medical Center
Tianjin University of Finance and Economics

Joint work with Weizhen Wang and David Annis
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Outline

1. Various Selection Procedures
2. NK-procedure Controls FWE for Simultaneous Tests in N
3. Some Simulation Results on Relative Efficiencies

Selection of the Best Population

- Indifference-zone approach (Bechhofer, 1954)
- Subset selection formulation (Gupta, 1956)
- Two-stage and sequential procedures

  (see Bechhofer, Santner, and Goldsman, 1995; Kim and Nelson, 2001; Chen and Kelton, 2005)
- These procedures select ONE of the best

  With multiple best populations, these procedures do not control the probability of including ALL best treatments.

Selection of ALL Best Population

- Singe-step test by Lam (1986)
- Unconstrained Multiple Comparison with the Best (UMCB) by Edward and Hsu (1983)
- Step-down procedure by Broström (1981) and Finner and Giani (1994)
- Acceptance set approach by Hayter (2007)
- Newman-Keuls (NK) procedure (restricted to MCB)

Balanced One-Way Layout Model with k Treatments

- Independent samples

$$\boxed{X_{ij} \sim N(\mu_i, \sigma^2),\ 1 \leq i \leq k, 1 \leq j \leq n} \qquad (1)$$

- Mean estimates $\hat{\mu}_i$
- Variance estimate $\hat{\sigma}^2 \sim \sigma^2 \chi^2_\nu / \nu, \quad \nu = k(n-1)$
- Studentized ordered statistics

$$Y_i = \sqrt{n}\hat{\mu}_{(i)}/\hat{\sigma}, 1 \leq i \leq k, \qquad (2)$$

where $(1), \ldots, (k)$ are random indices such that $Y_1 \leq Y_2 \leq \ldots \leq Y_k$.

Critical Values d, $|d|$, q

Suppose $Z_i \sim N(0,1)$ and $U \sim \sqrt{\chi^2_\nu/\nu}$ are independent. For any given α and ν, we define critical values:

- d_{k-1} as used in Dunnett's one-sided MCC method

$$P(\max_{1\le i\le k-1} Z_i - Z_k \le d_{k-1}U) = 1-\alpha \tag{3}$$

- $|d|_{k-1}$ as used in Dunnett's two-sided MCC method

$$P(\max_{1\le i\le k-1} |Z_i - Z_k| \le |d|_{k-1}U) = 1-\alpha \tag{4}$$

- q_k as used in Tukey's MCA method

$$P(\max_{1\le i,j\le k} |Z_i - Z_j| \le q_k U) = 1-\alpha \tag{5}$$

- Note that $d_{k-1} < |d|_{k-1} < q_k < q_{k+1}$

Critical Values (c, w)

Suppose $Z_i \sim N(0,1)$ and $U \sim \sqrt{\chi^2_\nu/\nu}$ are independent.

- For the BFG method $c_{k,k} = q_k$ and $c_{k,r}$ are defined by

$$P\begin{pmatrix} \max_{1\le a\le k-r} Z_a - \min_{k-r+1\le b\le k} Z_b \le c_{k,r}U \\ \max_{k-r+1\le i,j\le k} |Z_i - Z_j| \le c_{k,r}U \end{pmatrix} = 1-\alpha$$

- For the Hayter method, $w_{k,k} = q_k$ and $w_{k,r}$ are iteratively defined by

$$P\begin{pmatrix} \max_{1\le a\le k-r} Z_a - \min_{k-r+1\le b\le k} Z_b \le w_{k,r+1}U \\ \max_{k-r+1\le i,j\le k} |Z_i - Z_j| \le w_{k,r}U \end{pmatrix} = 1-\alpha$$

- $q_r < w_{k,r} \overset{?}{<} c_{k,r} < q_k$; $\overset{?}{<}$ holds when $w_{k,2} \le \ldots \le w_{k,k-1}$

Single Step Procedures

- Gupta (1956), Hsu (1984) CMCB

$$I = \{i : \; Y_i < Y_k - d_{k-1}\} \tag{6}$$

- Lam (1986)

$$I = \{i : \; Y_i < Y_k - q_k\} \tag{7}$$

- Edward and Hsu (1983) UMCB

$$\begin{aligned} G &= \{Y_j : \; Y_j > Y_k - |d|_{k-1}\} \\ I &= \{i : \; Y_i < \min G - |d|_{k-1}\} \end{aligned} \tag{8}$$

A General Description of Step-down Procedures

Step 1 Start with $T_k = Y_k - Y_1$. If $T_k \leq t_k$, then conclude that there is no inferior treatment and stop; otherwise, conclude that Y_1 is inferior and go to step 2.

Step 2 If $T_{k-1} = Y_k - Y_2 \leq t_{k-1}$, then stop; otherwise, conclude that Y_1, Y_2 are inferior and go to step 3.

$\vdots$

Step k-1 If $T_2 \leq t_2$, then conclude that $Y_1, \ldots, Y_{k-2}$ are inferior; otherwise, conclude that $Y_1, \ldots, Y_{k-1}$ are inferior.

$$\underline{Y_1 \leq \underline{Y_2 \leq \ldots \leq Y_{k-2} \leq \underline{Y_{k-1} \leq Y_k}}}$$

Three Step-down Procedures

- Broström (1981) and Finner and Giani (1994): $t_r = c_{k,r}$
- Hayter (2007): $t_r = w_{k,r}$
- Newman-Keuls (NK): $t_r = q_r$
- $BFG \prec Hayter \prec NK$

A Monotone Property

- Let N be the number of μ_i's being largest (wlog, 0)
- Hypotheses: $H_{0,r} : N \geq r$ vs $H_{A,r} : N \leq r - 1$

Theorem

For any $2 \leq r \leq k$, $T_r = Y_k - Y_{k-r+1}$ is stochastically largest at $\underline{\boldsymbol{\mu}}_r = (-\infty, \cdots, -\infty, 0, \cdots, 0,)$ (with r zero means) among $H_{0,r}$. Hence, the rejection region $\{T_r > q_r\}$ is a level-α test.

$$Y_1 \leq Y_2 \leq \ldots \leq Y_{k-r} \leq \underline{Y_{k-r+1} \leq \ldots \leq Y_k}$$

Strong Control of FWE

- To simultaneously test $\mathcal{B} = \{H_{0,r} : 2 \leq r \leq k\}$
- Assert $H_{A,r}$ if $\cap_{j=r}^{k} \{T_j = Y_k - Y_{k-j+1} > q_j\}$ occurs

$$\underline{Y_1 \leq \underline{Y_2 \leq \ldots \leq Y_{k-r} \leq \underline{Y_{k-r+1} \leq \ldots Y_k}}}$$

Theorem

The familywise error is controlled in the strong sense. More specifically, if $N(\underline{\mu}) \geq r + 1$, then

$$\boxed{P_{\underline{\mu}}(\text{assert } N \leq r) \leq \alpha.} \quad (9)$$

Sketch Proof of the Monotone Property

- Idea: stochastic ordering of random vector $(Y_{k-r+1}, -T_r)$ condition on $\hat{\sigma}$ (Kamae, krengel, and O'Brien, 1977)
- Let $p(u, v)$ and $q(u, v)$ be the distribution of $(Y_{k-r+1}, -T_r)$ condition on $\hat{\sigma}$ at $\underline{\boldsymbol{\mu}}_r$ and at $\underline{\boldsymbol{\mu}} \in H_{0,r}$, respectively.
- Fact 1: Marginal distributions satisfy $p_1(u) \prec q_1(u)$
- Fact 2: Conditional distributions satisfy

$$\boxed{p(v|u) \prec p(v|u') \prec q(v|u'), \ \forall u \leq u'} \quad (10)$$

- Therefore, $p(u, v) \prec q(u, v)$

Some Toy Examples with $k = 3$

Table: Inferior treatments identified

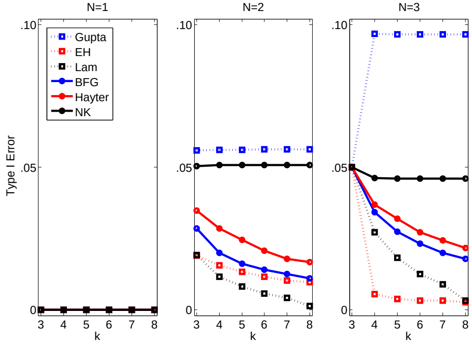
		Sample means		
Procedures	Cutoffs	$(0, 1, 4)$	$(0, 1.2, 4)$	$(0, 3.2, 3.3)$
Gupta	$d_2 = 2.710$	$\{1, 2\}$	$\{1, 2\}$	$\{1\}$
EH	$\lvert d\rvert_2 = 3.128$	$\emptyset$	$\emptyset$	$\{1\}$
Lam	$q_3 = 3.314$	$\{1\}$	$\{1\}$	$\emptyset$
BFG	$c_{3,2} = 3.105$	$\{1\}$	$\{1\}$	$\emptyset$
Hayter	$w_{3,2} = 2.968$	$\{1, 2\}$	$\{1\}$	$\emptyset$
NK	$q_2 = 2.772$	$\{1, 2\}$	$\{1, 2\}$	$\emptyset$

Simulation Setup

- $k = 3, \ldots, 8$
- $N = 1, 2, 3$
- $\sigma = 1$ known
- Inferior treatments equally spaced with maximum mean difference of 0.8
- Compare sample sizes needed so that

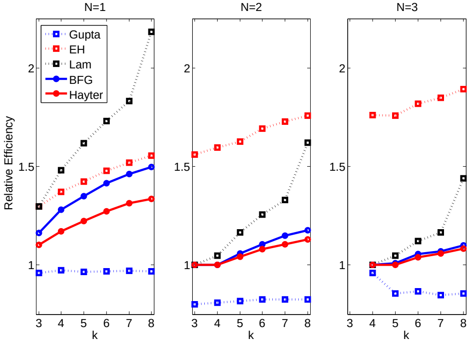
  P(Complete Correct Selection) = 0.8

Comparison of Type I Error Rate



- $\alpha \approx 0$ when N=1
- Gupta > NK > Hayter > BFG > (EH, Lam)

Relative Efficiencies of the NK Procedure



- Gupta > NK > Hayter > BFG > (EH, Lam)
- NK vs Hayter: $\approx 130\%, 115\%, 110\%$ for k=8

Possible Future Works

- Unbalance one-way layout
- ANCOVA, sample means with known dependent cov
- Group sequential setting
- Step-up tests for MCB

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Detailed Sample Sizes

Table: Sample sizes so that P(Complete Correct Selection) = 0.8

N	Procedures	k=3	k=4	k=5	k=6	k=7	k=8
1	Gupta	94	215	381	595	860	1167
	EH	127	303	562	909	1344	1875
	Lam	127	327	639	1065	1621	2632
	BFG	114	283	533	870	1294	1807
	Hayter	108	259	483	782	1163	1612
	NK	98	221	395	615	885	1206
2	Gupta	20	84	192	343	537	773
	EH	39	166	382	704	1123	1650
	Lam	25	109	274	523	865	1520
	BFG	25	104	249	460	747	1103
	Hayter	25	104	245	449	718	1060
	NK	25	104	235	416	650	938
3	Gupta	1	24	88	200	352	552
	EH	1	44	181	420	767	1221
	Lam	1	25	108	259	484	929
	BFG	1	25	104	244	444	709
	Hayter	1	25	103	240	439	699
	NK	1	25	103	231	415	645